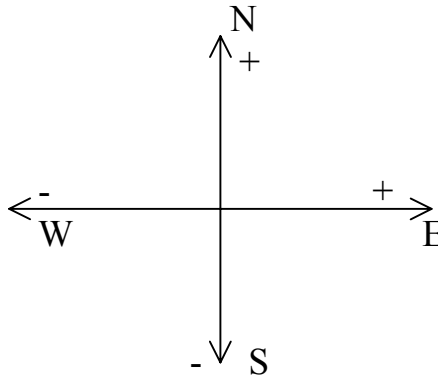


Vectors

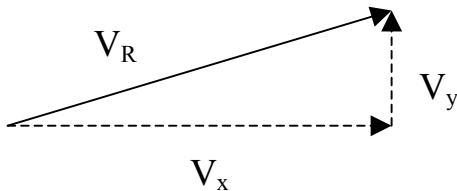
A vector has both magnitude and direction which can be represented by arrows where the length of the arrow is proportional to the magnitude, and the direction of the arrow is the direction of the vector.

Use the following coordinate system :



Vector quantities can be broken into components.

- x - component is the horizontal component
- y - component is the vertical component.



$$\sin \Theta = \frac{V_y}{V_R}$$

$$\cos \Theta = \frac{V_x}{V_R}$$

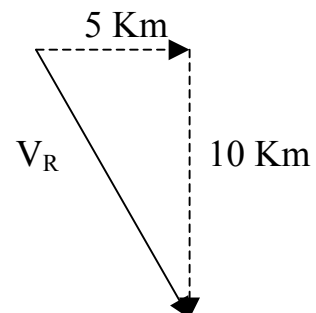
$$V_y = V_R \sin \Theta$$

$$V_x = V_R \cos \Theta$$

Vectors can also be drawn from x & y components.

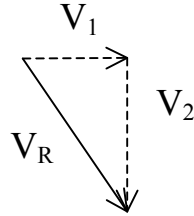
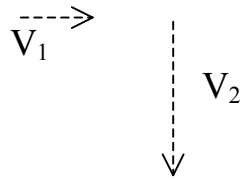
ie. $V_x = +5 \text{ Km}$

$V_y = -10 \text{ Km}$



Vector Addition :

a) Graphically – Always add “tip to “tail”



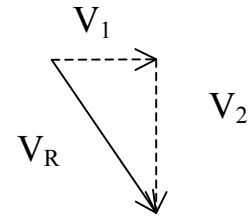
$$V_R = V_1 + V_2$$

b) By Components

$$V_1 = 5 \text{ Km E}$$

$$V_2 = 10 \text{ Km S}$$

	x	y
V_1	5	0
V_2	0	-10
	5	-10



To find the **magnitude**, use the Pythagorean Theorem

$$c^2 = a^2 + b^2 \quad V_R^2 = V_1^2 + V_2^2 = 5^2 + (-10)^2$$

$$V_R^2 = 125 \quad V_R = 11 \text{ Km}$$

To find the **direction**, use trigonometry

$$\text{Tan } \Theta = \text{opposite} / \text{adjacent}$$

$$\text{Tan } \Theta = V_2 / V_1$$

$$\text{Tan } \Theta = -10 / 5$$

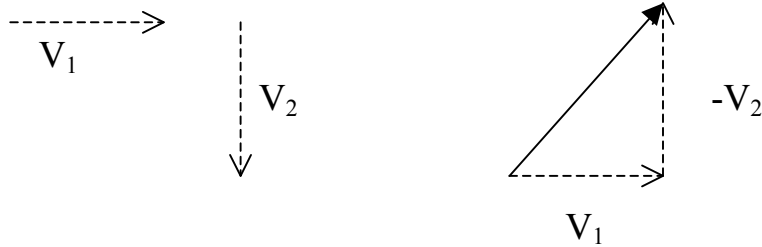
$$\Theta = -63^\circ \text{ or } 63^\circ \text{ S of E}$$

Check your theta
with your original
sketch !!

Vector Subtraction

a) Graphically – flip the direction on the 2nd vector

$$V_1 - V_2 = V_1 + (-V_2)$$



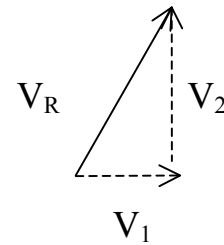
** Recall that $\Delta V_f - V_o$

b) By component :

$$V_1 = 5 \text{ Km E}$$

$$V_2 = 10 \text{ Km S}$$

	x	y
V_1	5	0
V_2	0	+10
	5	+10



$$c^2 = a^2 + b^2$$

$$V_R^2 = V_1^2 + V_2^2 = 5^2 + 10^2$$

$$V_R^2 = 125$$

$$V_R = 11 \text{ Km}$$

Tan Θ = opposite / adjacent

$$\text{Tan } \Theta = V_2 / V_1$$

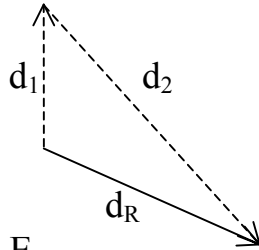
$$\text{Tan } \Theta = 10 / 5$$

$$\Theta = 63^\circ \text{ or } 63^\circ \text{ E}$$

Check your theta
with your original
sketch !!

Example One : An explorer walks 22.0 Km North, then 47.0 Km 60 ° South of East. What is his resultant displacement?

Draw rough sketch :



Vector 1 : d_1 : $\Theta = 90^\circ \text{ E}$

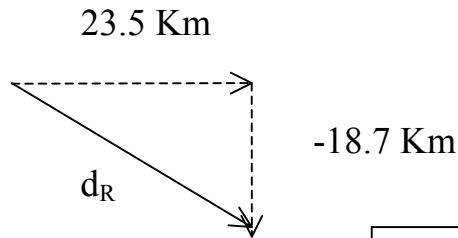
$$\begin{aligned} d_x &= d_1 \cos \Theta &= 22.0 \text{ Km} \cos 90^\circ &= 0 \text{ Km} \\ d_y &= d_1 \sin \Theta &= 22.0 \text{ Km} \sin 90^\circ &= 22.0 \text{ Km} \end{aligned}$$

Vector 2 : d_2 : $\Theta = 300^\circ \text{ E}$

$$\begin{aligned} d_x &= d_2 \cos \Theta &= 47.0 \text{ Km} \cos 300^\circ &= 23.5 \text{ Km} \\ d_y &= d_2 \sin \Theta &= 47.0 \text{ Km} \sin 300^\circ &= -40.7 \text{ Km} \end{aligned}$$

ALWAYS USE theta from EAST for your component calculations!!

	x	y
d_1 :	0	22.0
d_2 :	23.5	-40.7
d_R :	23.5	-18.7



Check to see that d_R is the same as in your original sketch !!

$$\begin{aligned} c^2 &= a^2 + b^2 & d_R^2 &= d_1^2 + d_2^2 = 23.5^2 + (-18.7)^2 \\ d_R^2 &= 901.94 & d_R &= 30.0 \text{ Km} \end{aligned}$$

Tan Θ = opposite / adjacent

$$\text{Tan } \Theta = y / x$$

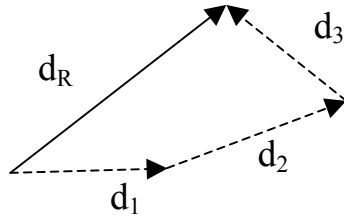
$$\text{Tan } \Theta = -18.7 / 23.5$$

$$\Theta = -38.5^\circ \text{ S of E or } 321^\circ$$

Check your theta with your original sketch !!

Example Two : Determine the resultant displacement when a smelly dog chases a bunny rabbit 25 m due East, 34 m 70 ° East of North, and 30 m at 45 ° West of North.

Rough sketch :



ALWAYS USE
theta from EAST
for your component
calculations!!

Vector 1 : d_1 : $\Theta = 0^\circ$ E

$$d_x = d_1 \cos \Theta = 25.0 \text{ m} \cos 0^\circ = 25 \text{ m}$$

$$d_y = d_1 \sin \Theta = 25.0 \text{ m} \sin 0^\circ = 0 \text{ m}$$

Vector 2 : d_2 : $\Theta = 20^\circ$ E

$$d_x = d_2 \cos \Theta = 34 \text{ m} \cos 20^\circ = 31.94 \text{ m}$$

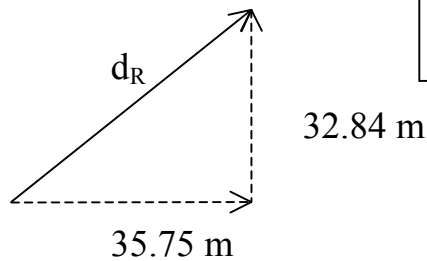
$$d_y = d_2 \sin \Theta = 34 \text{ m} \sin 20^\circ = 11.63 \text{ m}$$

Vector 3 : d_3 : $\Theta = 135^\circ$ E

$$d_x = d_3 \cos \Theta = 30 \text{ m} \cos 135^\circ = -21.21 \text{ m}$$

$$d_y = d_3 \sin \Theta = 30 \text{ m} \sin 135^\circ = 21.21 \text{ m}$$

	x	y
d_1 :	25	0
d_2 :	31.94	11.63
d_3 :	<u>-21.21</u>	<u>21.21</u>
d_R :	35.75	32.84



Check to see that d_R is
the same as in your
original sketch !!

$$c^2 = a^2 + b^2 \quad d_R^2 = d_1^2 + d_2^2 = 35.75^2 + 32.84^2$$
$$d_R^2 = 2356.52 \quad d_R = 48.54 \text{ m} = 49 \text{ m}$$

Tan Θ = opposite / adjacent

$$\text{Tan } \Theta = y / x$$

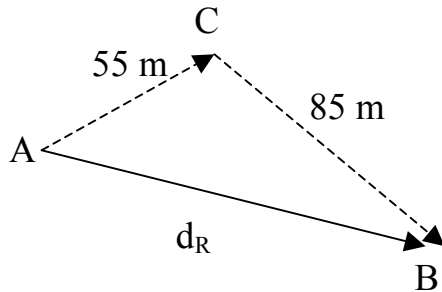
$$\text{Tan } \Theta = 32.84 / 35.75$$

$$\Theta = 42.57^\circ \text{ E} = 43^\circ \text{ E}$$

Check your theta
with your original
sketch !!

Example Three : Find the displacement of a three legged rat who limps 55 m at 30° North of East, then 85 m 35° South of East.

Rough sketch :



Since this is a three sided triangle, we can use the Law of Sines and Cosines OR we can use the x, y component method.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2 ab \cos C \\ &= 85^2 + 55^2 - 2 (55)(85) \cos 115^\circ \\ d_R = c &= 119 \end{aligned}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\sin A = \frac{(\sin C)(a)}{c} = \frac{(\sin 115^\circ)(85)}{119} = 40^\circ$$

$$\therefore \Theta = 40^\circ - 30^\circ = 10^\circ \text{ South of East}$$

Try it using the component method to check you answer!

